Systematic Patterns in the E1 Decays in ^{21}Ne and ^{23}Na: A Means of Identifying High Spin States?

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The E1 decays from members of the lowest $K^\pi = 1/2^-$ bands in ^{21}Ne and ^{23}Na are shown to exhibit remarkable, but dissimilar, patterns, which can be reproduced with a simple two-parameter expression. This may have useful application in identifying high-spin members of the $K^\pi = 1/2^-$ and $3/2^+$ bands in both nuclei.

Electric dipole (E1) transitions in light nuclei have received little theoretical attention because they are so difficult to calculate. Spurious centre of mass motion must be carefully treated, and when this has been done, one often finds selection rules [1] which prohibit the transition between the dominant components of the initial and final wavefunctions. A famous case is ^{19}F: the ground state band is dominated by the (60) irreducible representation of SU(3), while the low-lying negative parity 4p – 1h band is dominantly (81). The E1 operator transforms under SU(3) as (10) and clearly cannot connect these components. E1 transitions therefore depend critically on very small configurational admixtures to the wavefunctions, and it is not surprising that they are small and span a wide, often unsystematic, range of strengths.

Some time ago, however, a remarkable "pattern" of E1 transitions from the lowest $K^\pi = 1/2^-$ band of ^{23}Na, based on the #4 Nilsson orbital with quantum numbers $1/2^-$ [101] was noted [2], and agreement with a very simple Nilsson model calculation was obtained. This calculation had certain deficiencies, such as difficulties with constructing non-spurious states and treating hole configurations correctly [3] and the agreement, though gratifying, was still rather unconvincing. Moreover, the "pattern" emerged from complicated band mixing procedures which were necessarily most unenlightening. The E1 pattern observed was quite striking: when the E_γ^3 dependence of the transition rates was removed, a spin J state belonging to the $1/2^-$ band was observed to decay preferentially to a spin $J+1$ state in the ground state ($K^\pi = 3/2^+$) band. These results were greatly at variance with the simple rotational model (Alaga model) where the reduced transition strengths should simply be proportional to the appropriate Clebsch-Gordan coefficients squared. The discrepancy between experiment and the Alaga model is illustrated in columns 5 and 7 for Table 1.

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Table 1. Relative reduced E1 strengths in ^{21}Ne and ^{23}Na.

J^π initial	final	^{21}Ne Expt	GAM-1	^{23}Na Expt	GAM-2	Alaga
$1/2^-$	$3/2^+$	0	0 *	100	100	100
$3/2^-$	$3/2^+$	100	100	3	3	40
$3/2^-$	$5/2^+$	0	0 *	97	97	60
$5/2^-$	$3/2^+$	33	55	5	6	6
$5/2^-$	$5/2^+$	67	44	5	9	45
$5/2^-$	$7/2^+$	<10	1	90	85	48
$7/2^-$	$5/2^+$	< 2	22	40	35	10
$7/2^-$	$7/2^+$	6	1	<5	0	48
$7/2^-$	$9/2^+$	94	77	60	65	42
$9/2^-$	$7/2^+$	64	62	3	5	13
$9/2^-$	$9/2^+$	36	37	8	13	48
$9/2^-$	$11/2^+$	< 5	1	89	82	39
$11/2^-$	$9/2^+$		24		37	15
$11/2^-$	$11/2^+$		2		0	48
$11/2^-$	$13/2^+$		74		63	37
$13/2^-$	$11/2^+$		65		5	16
$13/2^-$	$13/2^+$		34		15	49
$13/2^-$	$15/2^+$		1		80	35
$15/2^-$	$13/2^+$		25		37	17
$15/2^-$	$15/2^+$		2		0	49
$15/2^-$	$17/2^+$		73		63	34

GAM-1 used $m_1 = -\sqrt{6}$, $m_2 = -\sqrt{3}$ to reproduce the starred vanishing transitions.
GAM-2 used $m_1 = 2.0$, $m_2 = -0.7$ to reproduce the transitions in ^{23}Na.
Experimental values from Refs. [2] (^{23}Na) and [4, 5] (^{21}Ne).

Recent experiments [4] on the very similar nucleus ^{21}Ne have extended the level and decay scheme of the corresponding negative parity band up to $J^\pi = 9/2^-$. The results of previous [5] and the above experiments have established the following states as members of the $K^\pi = 1/2^-$ band (energies in keV);

$1/2^-$, 2790; $3/2^-$, 3663; $5/2^-$, 3886; $7/2^-$, 5823; and $9/2^-$, 6034. Table 1 also shows the experimentally deduced relative E1 strengths for the transitions from these states to those in the ground state rotational band. It is again evident that there is a marked discrepancy from the predictions of the Alaga model, but it is interesting to note that they are quite different from the analogous decays in ^{23}Na. Neither do they show as clear-cut a pattern, although certain systematic features can certainly still be observed.

Such regular deviations from the Alaga rule must imply an interference effect. The most likely source is provided by mixing of Nilsson orbitals into the ground state band, especially of those with $K^\pi = 1/2^+$. There are two $K^\pi = 1/2^+$ bands which can mix into the ground state band of either ^{21}Ne or ^{23}Na: the $1/2^+$ [220] (orbit #6) and $1/2^+$ [211] (orbit #9). Of these two, only the one based on orbit #6 can be reached via a one-body operator from the negative parity band. The rotational model then predicts that the E1 transition rate to the band-mixed positive parity states is given by

$$B(\mathrm{E1}) = |(J\,1/2\,1\,1\,|\,J'\,3/2) + m_1(J\,1/2\,1\,0\,J'\,|\,1/2) + (-)^{J'-1/2} m_2 (J\,1/2\,1\,-1\,|\,J'-1/2)|^2 \quad (1)$$

where J and J' are the spins of the initial and final states, respectively; m_1 and m_2 are parameters which can be related to the degree of mixing between the $1/2^+$ and $3/2^+$ bands. Note that Eq. (1) assumes that they are constant as one proceeds up the band: this is equivalent to assuming that the mixing is the same for all members of the ground state band.

One can now proceed in two ways: the matrix elements m_1 and m_2 may be calculated from a nuclear model (such as the Nilsson model) or they may be taken to be free parameters. We first show that Eq. (1) gives a very good description of the E1 transitions in both ^{21}Ne and ^{23}Na if m_1 and m_2 are properly chosen.

We note that there are two extremely weak – almost vanishingly small – E1 transitions in ^{21}Ne; namely, $1/2^- \to 3/2^+$ and $3/2^- \to 3/2^+$, and we may therefore approximate these by letting them vanish identically. We therefore obtain, from Eq. (1), a set of equations whose solution yields $m_1 = -\sqrt{6}$ and $m_2 = -\sqrt{3}$, and Eq. (1) may now be used to predict the relative reduced E1 rates from the higher spin states. These predictions are given in column 4 of Table 1 under the heading GAM (generalized adiabatic model), and it is clear that reasonable agreement is obtained for the decays from the $J^\pi = 5/2^-$, $7/2^-$ and $9/2^-$ states.

A similar analysis may be carried out for the E1 decays in ^{23}Na. The two input parameters were obtained by requiring that the known transitions from low-lying states (up to $J^\pi = 5/2^-$) be reproduced; we then obtained $m_1 = 2.0$ and $m_2 = -0.7$. The relative reduced E1 strengths calculated using these parameters in Eq. (1) are given under the heading GAM in column 6 of Table 1, and good agreement with experiment for the higher lying states is obtained.

The parameter searches undertaken were not exhaustive and it was found that more than one set of m_1 and m_2 gave reasonable agreement with the experimental information. One should like to relate these quantities with those obtained from a more detailed calculation. The parameters m_1 and m_2 defined above may be written in terms of Nilsson's $G_{E\lambda}$ and $b_{E\lambda}$ [see e. g. Eq. (35) of Ref. [6]] via

$$m_1 = \frac{\beta\, G_{\mathrm{E1}}\ (1/2 \to 1/2)}{\alpha\, G_{\mathrm{E1}}\ (1/2 \to 3/2)}$$

$$m_2 = m_1 \cdot b_{\mathrm{E1}}\,(1/2 \to 1/2)$$

where α and β are the amplitudes of $K = 3/2$ and $K = 1/2$, respectively, in the ground state band.

For a reasonable value for the mixing of the $K^\pi = 1/2^+$ into the $K^\pi = 3/2^+$ band (say $\alpha^2 = 0.8$, $\beta^2 = 0.2$) it is not possible to find a value for the deformation η (assumed positive and the same for all intrinsic states) which can reproduce the observed pattern in either of the two nuclei as well as is done by the GAM. In ^{23}Na, one can, it is true, enhance the $J^- \to (J+1)$ transition over the Alaga estimate but not to the extent that is experimentally observed. The very strange behaviour in ^{21}Ne cannot be reproduced at all. Yet the striking success of our simple parameterization must have a microscopic basis. It is clearly worthwhile to search for these "patterns" in other light nuclei if the phenomenon is to be better understood.

A large number of high spin states have recently been proposed in ^{23}Na from their strong population in heavy ion reactions [7, 8] but the data are somewhat in conflict and it has proved very difficult to assign levels to rotational bands. The success of our model for E1 transitions for low spin states suggests that a study of the γ-ray decay of these new high spin states may be used to identify members of the $K^{\pi} = 1/2^-$ and $3/2^+$ bands. Very low energy γ-rays (as was noted in Ref. [2]) are then indicative of a $J^- \rightarrow (J+1)$ transition, and to this end, we also present in Table 1 the expected reduced E1 strengths from the $11/2^-$, $13/2^-$ and $15/2^-$ states in both ^{21}Ne and ^{23}Na. We emphasize, however, that these "patterns" are not microscopically understood; nevertheless, it would be most unexpected if they did not persist to higher J states.

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